Trajectorial Otto calculus

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Abstract

We revisit the variational characterization of diffusion as entropic gradient flux, established by Jordan, Kinderlehrer, and Otto in [1], and provide for it a probabilistic interpretation based on stochastic calculus. It was shown in [1] that, for diffusions of Langevin-Smoluchowski type, the Fokker-Planck probability density flow minimizes the rate of relative entropy dissipation, as measured by the distance traveled in the ambient space of probability measures with finite second moments, in terms of the quadratic Wasserstein metric. We obtain novel, stochastic-process versions of these features, valid along almost every trajectory of the diffusive motion in both the forward and, most transparently, the backward, directions of time, using a very direct perturbation analysis. By averaging our trajectorial results with respect to the underlying measure on path space, we establish the minimum rate of entropy dissipation along the Fokker-Planck flow and measure exactly the deviation from this minimum that corresponds to any given perturbation. As a bonus of the perturbation analysis, we derive the so-called HWI inequality relating relative entropy (H), Wasserstein distance (W) and relative Fisher information (I).

Joint work with I. Karatzas and B. Tschiderer.

References

[1] R. Jordan, D. Kinderlehrer, and F. Otto (1998) The variational formula of the Fokker-Planck equation. *SIAM journal on mathematical analysis* **29**, no. 1, 1–17.