13TH EUROPEAN SUMMER SCHOOL IN FINANCIAL MATHEMATICS | 04.09.2020

DEEP LEARNING TECHNIQUES

In the context of economic capital and cash flow projections

Götz Cypra, Alexander Filler | UNIQA Insurance Group

Agenda

Background and Motivation

Background and Motivation
Task

Task: Analyse and optimise the asset allocation given the metrics of Solvency II

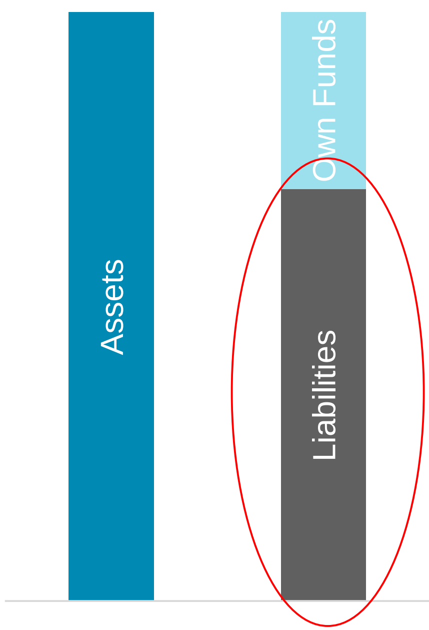
- The main challenges to analyse and optimise the asset allocation of a traditional life insurance book with options and guarantees arise from:
  i. The determination of the own funds requires a stochastic calculation / valuation for each capital market situation and asset allocation
  ii. The determination of the required risk capital (SCR) requires the simulations of changes of own funds (according step ii)
  iii. Each variation of asset allocation needs a new derivation of step i. and ii.
- Steps i. und ii. typically need several weeks.
- Exhaustive optimization of the asset allocation with respect to market risk of both sides of the balance sheet are usually not possible due to time constraints
- In practice only a few sensitivities of the asset allocation are calculated (partly only approximative) and numbers like own funds and risk capital reported
- Hence the multi dimensional space of reasonable measures within the strategic asset allocation or steering of assets cannot be analysed deeply.

For analysis and optimization of the asset allocation we need a „smart“ shortcut

Background and Motivation
Insights to risk capital calculation: Own Funds

i. The determination of the own funds requires a stochastic calculation / valuation for each capital market situation and asset allocation

Balance Sheet



Own Funds are the difference between market value of assets and (mark-to-model) market value of liabilities.

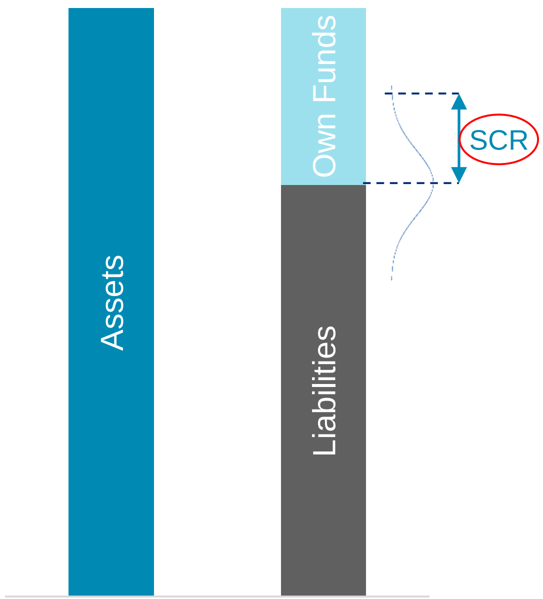
The liabilities of traditional life business need amongst other parameters:

- risk neutral Monte Carlo simulations of market variables such as example given interest rates, equity returns, etc.
- asset portfolio that is backing the liabilities
- Cash flows from / to policy holder (among other) given the local regulatory and accounting rules

Background and Motivation
Insights to risk capital calculation: SCR

ii. The determination of the required risk capital (SCR) requires the simulations of changes of own funds

Balance Sheet



The risk capital needs simulations of own funds. For market risk this requires

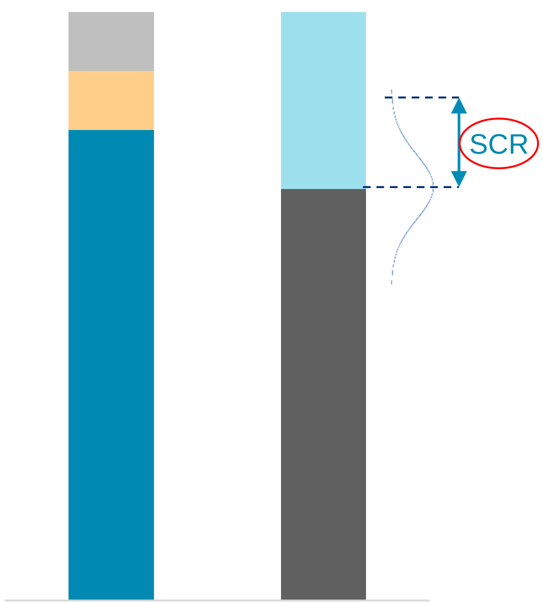
- ✓ real world simulations of market variables such as example given interest rates, equity returns, etc
- ✓ Simulations of assets dependent on these real world simulations
- ϟ Simulations of liabilities dependent on these real world simulations, and thus, theoretically, for each real world simulation a set of risk neutral simulations -> nested simulations

To overcome the problem of nested simulations approximation techniques are typically used

Background and Motivation
Insights to risk capital calculation: SCR dependent on asset allocation

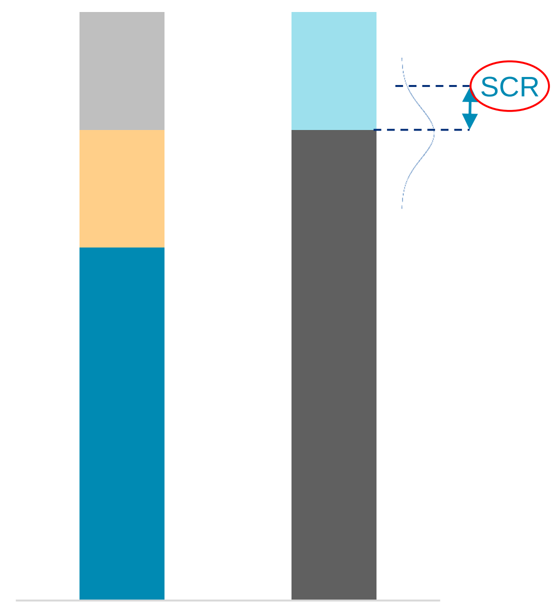
iii. Each variation of asset allocation needs a new derivation of step i. and ii.

Balance Sheet



Modifying the balance sheet, by e.g. changing the asset portfolio, needs a full recalculation of the previous steps

Balance Sheet



The inclusion of changes of the asset portfolio in the before mentioned approximation techniques is highly non trivial

Background and Motivation
Insights to risk capital calculation

The problem of evaluating liabilities, that are dependent on assets, is not new, but due to ist complexity, evolvement takes time. Nevertheless, different techniques already exist, where

- Replicating portfolios
- Least Squares Monte Carlo

are the most widely used, with different advantages / disadvantages for determining the SCR.

The additional requirement of including different asset allocations is definetly a challenge for these methods.

Within the last years, machine learning has been in the focus of academics again, such that it becomes more attractive to try these methods also for our problem here.

Background and Motivation

Abstract formulation of the problem

- Target variable Y shall be explained by explanatory variables $x_1, \dots, x_n$ via functional dependency (x_i are called „risk factors“)
- Examples of combinations of target variables and explanatory variables:
  - Solvency II – own funds at $t = 0$ explained by market conditions at $t = 0$ and/ or asset allocations-parameter
  - Own funds at $t = j$, $j > 0$, market conditions at $t = 1, \dots, j$
  - Cash Flows of liabilities at $t = j$ and relevant explanatory variables

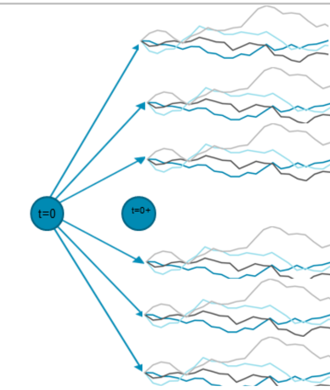
Advantages of Deep Learning methods versus traditional methods

1. Better reflection of cross effects
2. Easier and more canonical adaption to constraints or not smooth effects (e.g. if target variable must not be negative or the function is kinked)
3. Dynamic learning possible

The three aspects will be illustrated by examples in the next slides

Approach

Training data



Outer scenarios | inner scenarios

- Many initial stresses of the risk factors of which the impact on the own funds shall be explained
- **Concrete**:
  – Market risk factors (shape of initial interest curve, spread environments, market values of equities / real estate)
  – Parameter of initial asset allocation

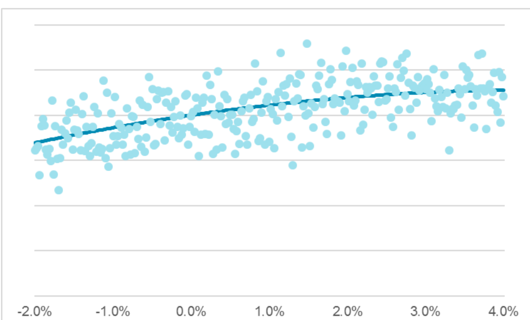
Risk neutral simulations based on the initial market environment per outer stress. Here only using a view simulations for the evaluation of the own funds

- **Idea**:
  – Collect for as many position as possible within the high dimension risk factor space information for the dependency between risk factors and own funds
  – Information are inaccurate (high variance) but unbiased
  – NN ideally extracts the trend without explaining too much of the noise

Background and Motivation

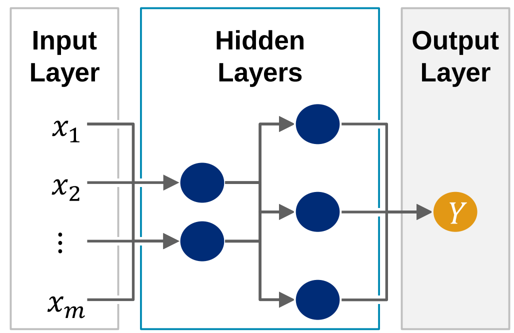
Current approach at UNIQA

- Least Squares Monte Carlo (LSMC) for SCR calculations
- Concrete: polynomial explanation of the dependency of own funds on risk factors by using model selection procedures
- Focus lies on SCR calculations – changes of asset allocations not within focus



Desired improvement by neural networks

- Better reflection of asset allocations
- Capability of complex dependency structures
- More flexibility: including new and different inputs by training, especially asset overlapping KPIs
- Continuous training on free server capacities and regular runs
- Increasing longevity by capability of dealing with complex dependency structures and additional trainings



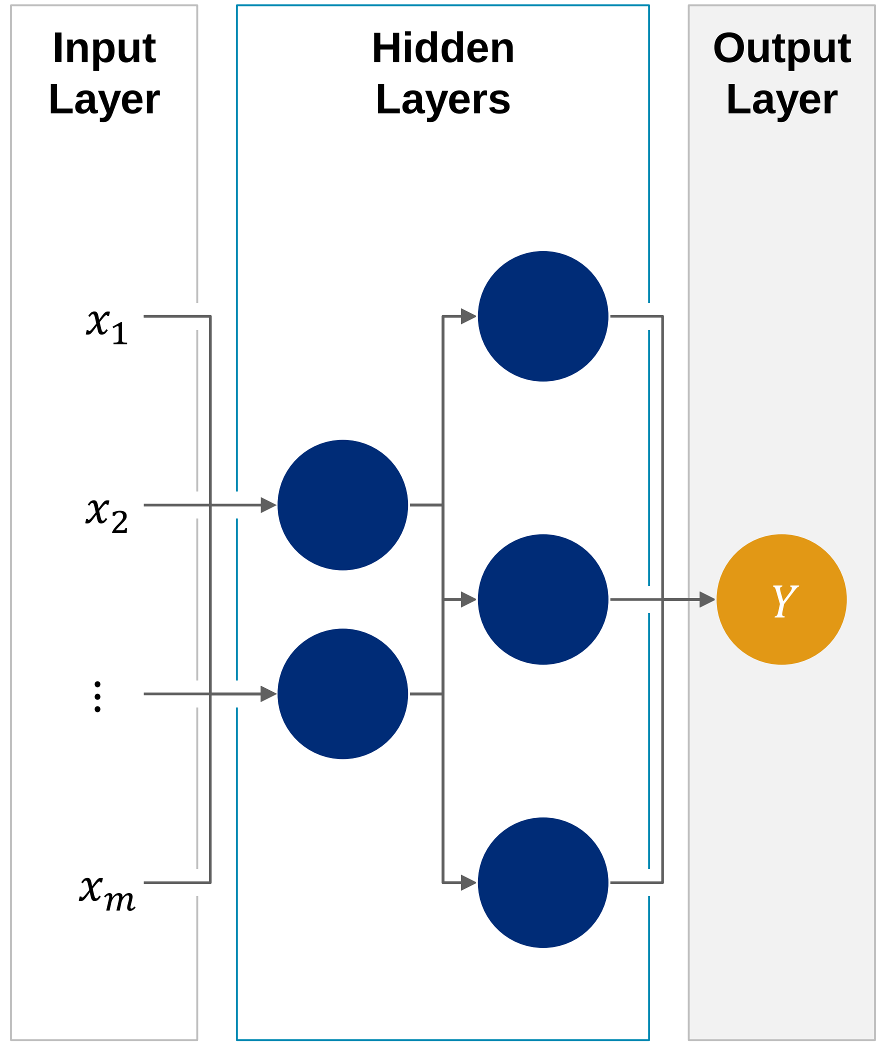
The Theory of Neural Networks

User Perspective

The Theory of Neural Networks
Overview

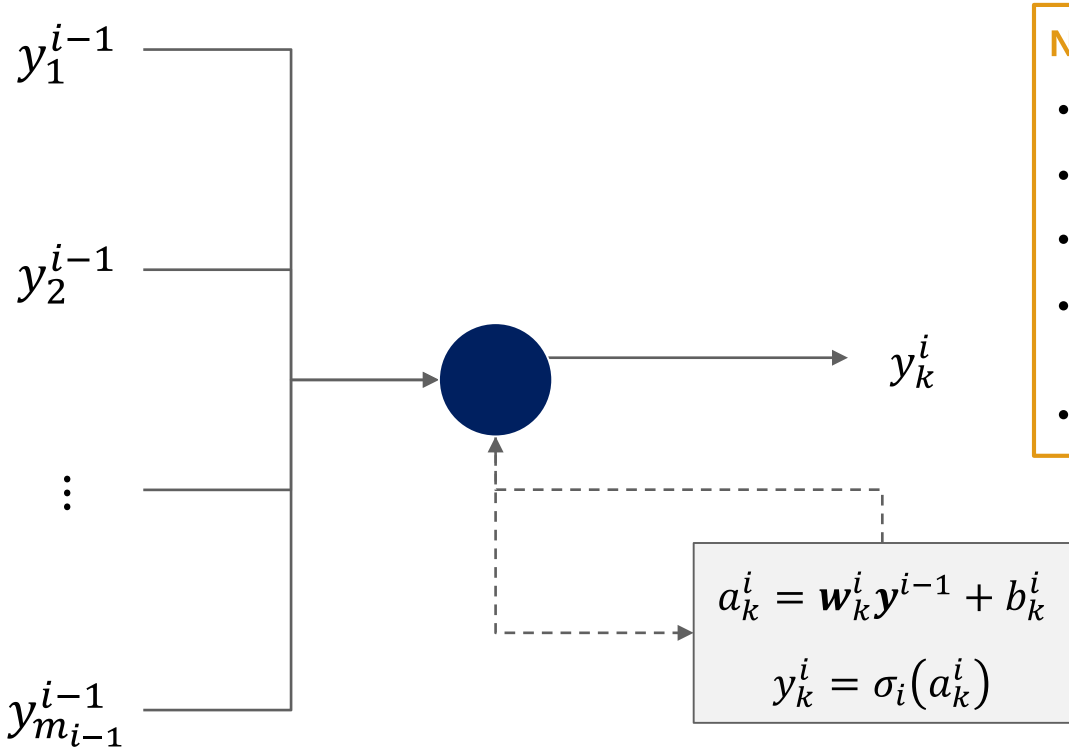
- A neural network (NN) defines a function for signal processing
- Input and output signal can have any dimension
- The core computing operations take place within the „neuros“
- Traditional applications are text, image and language recognition
- We use the neural networks only in the context of classic regression:

$$Y = NN(x_1, \dots, x_m) + \varepsilon$$



The Theory of Neural Networks
The Neuron

Calculation step wihin a neuron:



Notation

- y_k^i: Output of neuron k in layer i
- $y_k^0 := x_k$
- m_i: Number of neurons in layer i
- $\boldsymbol{w}_k^i, b_k^i$: Parameter of neuron k in layer i
- σ_i: activation function of layer i

The basic calculations within each single neuron combined with the given structure of the net make the NN surprisingly manageable.

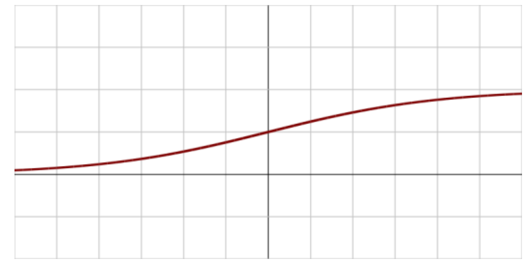
The Theory of Neural Networks
Activation Function

- Activation function are used to reflect non linearities:

$$\boldsymbol{y}^i = \sigma_i\left(\boldsymbol{W}^i\,\sigma_{i-1}\left(\boldsymbol{W}^{i-1}\boldsymbol{y}^{i-2} + \boldsymbol{b}^{i-1}\right) + \boldsymbol{b}^i\right)$$

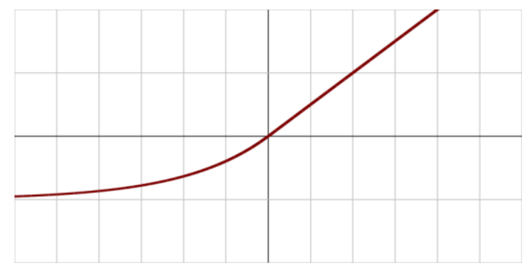
- For each layer different activation functions can be used
- The activation function of the output layer has significant impact on the behaviour of the output signal
- For a function approximation we normally use the identity (linear activation function)

Sigmoid



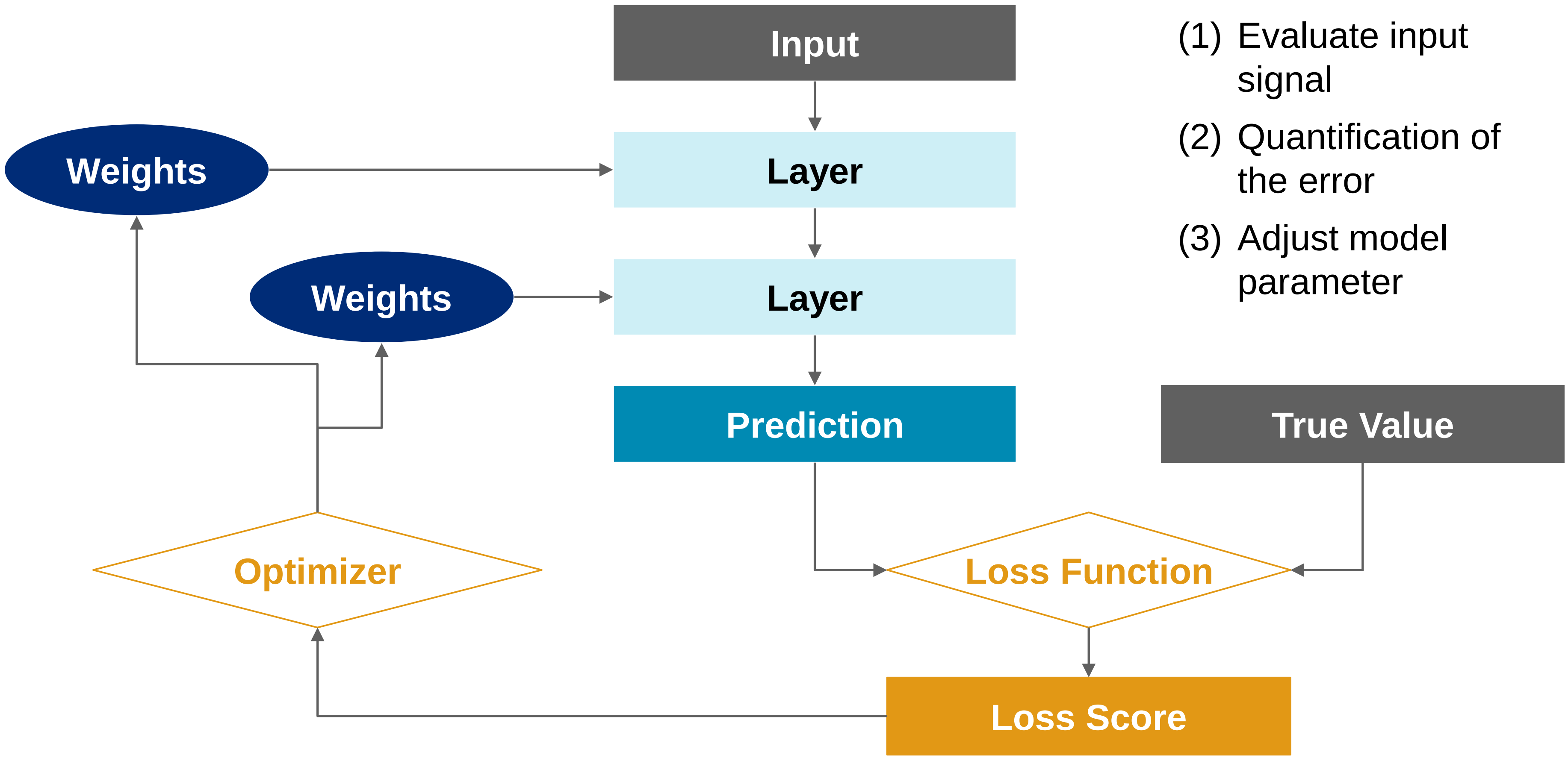
ReLU

ELU



The activations functions are the integral part to generate complex dependency structures with a neural network

The Theory of Neural Networks
The Training Cycle



(1) Evaluate input signal
(2) Quantification of the error
(3) Adjust model parameter

The calibration of the model parameter requires multiple runs of the training cycle

The Theory of Neural Networks
Hyperparameter

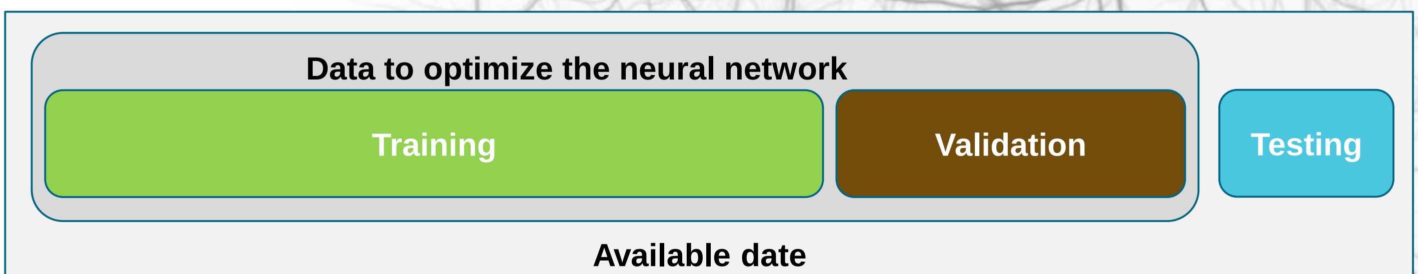
Many hyperparameter cannot be considered in the training i.e. they cannot be adapted by the training:

Structure of the NN

- Number of layers
- Number of neurons in each layer
- Activation function
- Dependency structure

Process of leaning

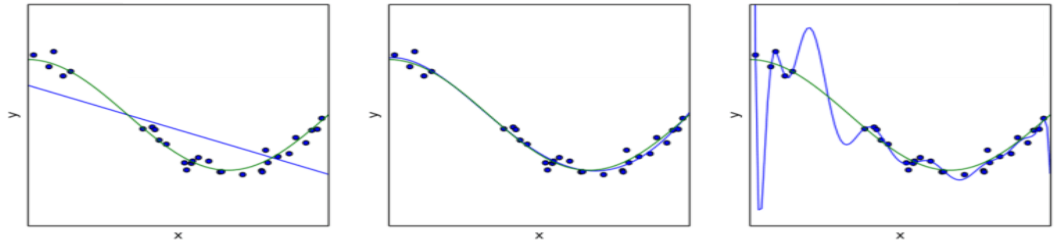
- Loss function
- Optimizer
- Step size / learning rate
- Batch size
- Number of epochs

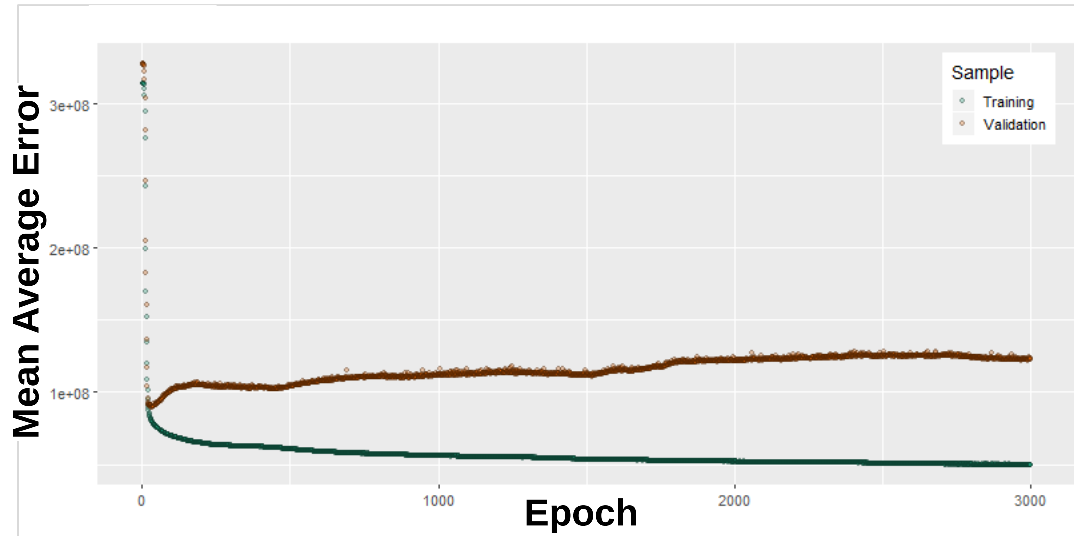


The „best“ structure of the NN is determined by evaluating the validation sample with the trained NN.

NN are vulnerable to overffiting: Commen approaches for regularization:

- Weight regularization
- Dropout
- Early stopping
- Reduction of complexity of the NN (layers, number of neurons)





The ability to represent complex dependencies makes the NN vulnerable to overfitting.

Analysis of SAA under Solvency II
Risk capital numbers

Case Study

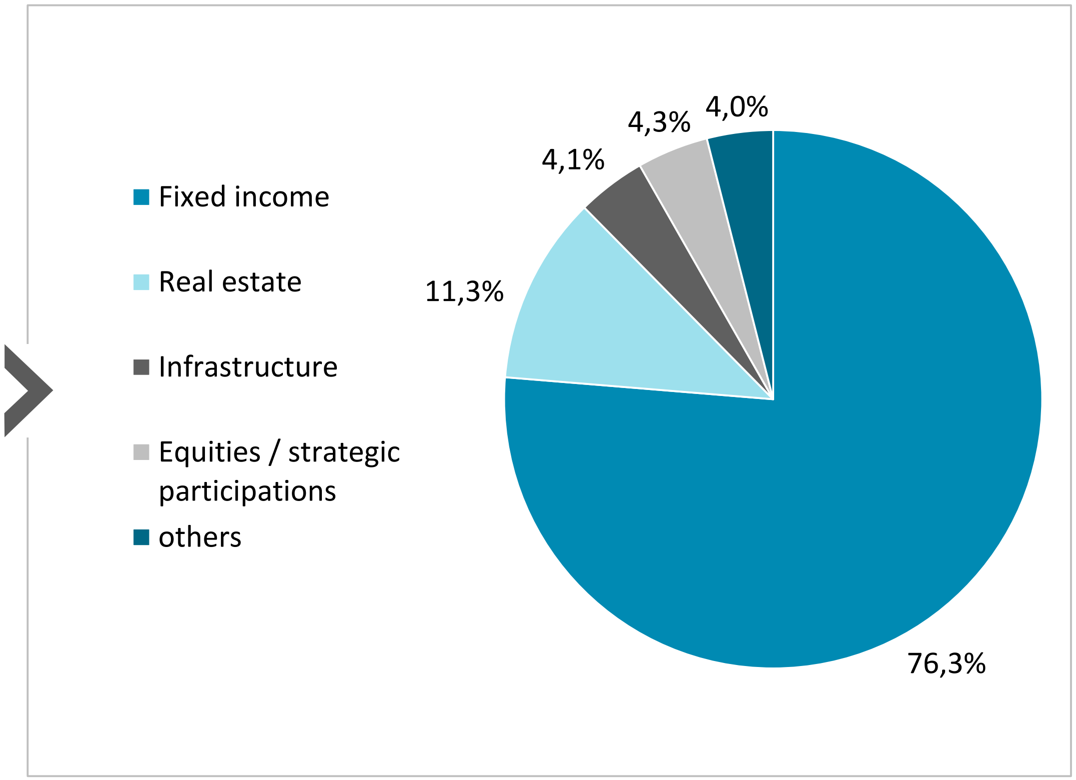
Background and motivation

Motivation

How do own funds and risk capital of classical life incurance change dependent on asset allocation?

Asset allocation

- Duration Assets: ca. 10
- Averag rating: A-
- SAA approach:
  – Sustainable and permanent earnings needed for guarantees
  – Having enough cash at any time
  – No substantial increase of risk



Background and motivation
Questions

Estimation of the effects of typical measures within strategic asset allocations for traditional with profit life insurance business:

1 Reduce duration gap:

- Duration gap after increase of asset duration in interest rate scenarios?

2 Higher spreads and lower duration gap:

- Lower risk with higher returns?

3 Increase of real estates within portfolio by reducing fixed income portfolio:

- Are properties the better asset class?

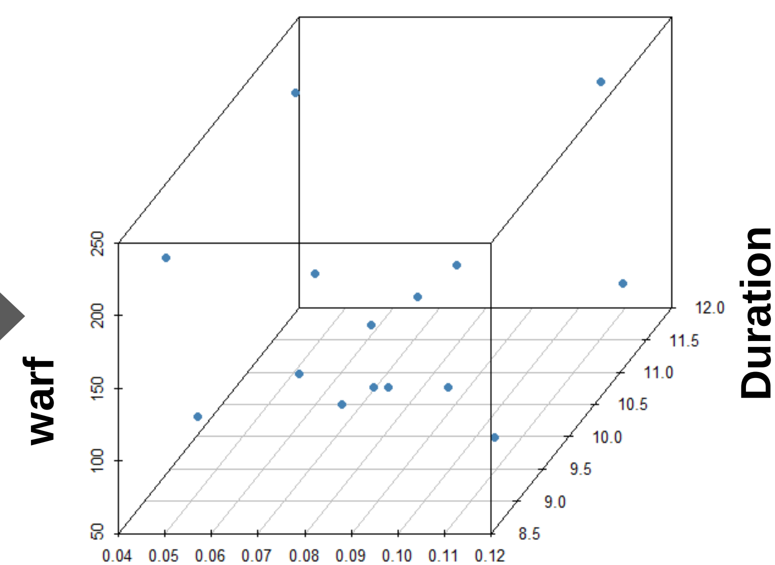
4 Increase of real estates and equities by reducing fixed income portfolio:

- Can hidden reserves of real estates compensate the risk coming from equities compensate?

Approach

Asset allocations

- Variations within three dimension
  - Allocation: Fixed Income vs real estates: from -5%- to +5%-Punkte
  - Average duration: -2 bis +3 Jahre
  - Rating: BBB to A+ (warf)
  - 130 combinations as a result
- Reallocation proportional or avoiding big changes within hidden reserves
- Automation (with python):
  - Reallocation and new Prophet Asset Tables
  - New Tables for Management Ruless
  - Parameters of asset allocations (portion of assets, duration, warf, hidden reserves etc) as inputs for the neural net
  - Adaption of market values corresponding to stresses of risk factors (Market Value Adjustment)

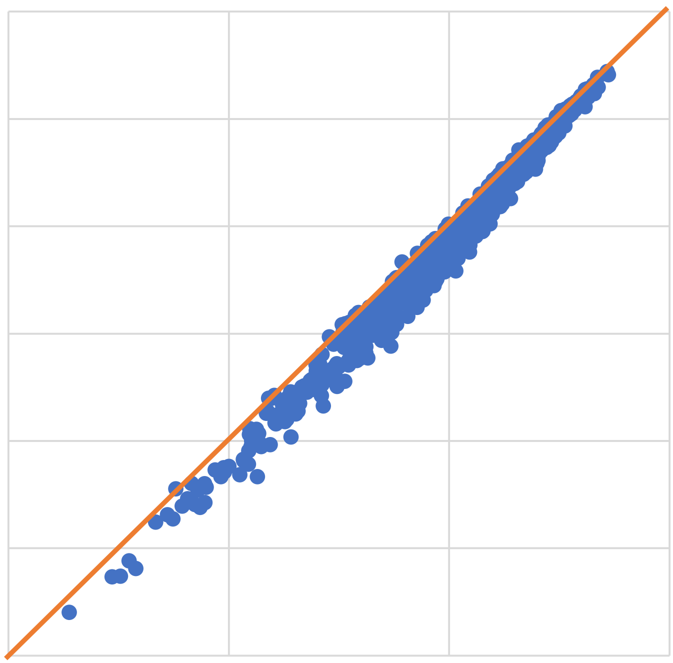


Realloaction FI vs. Real Estate

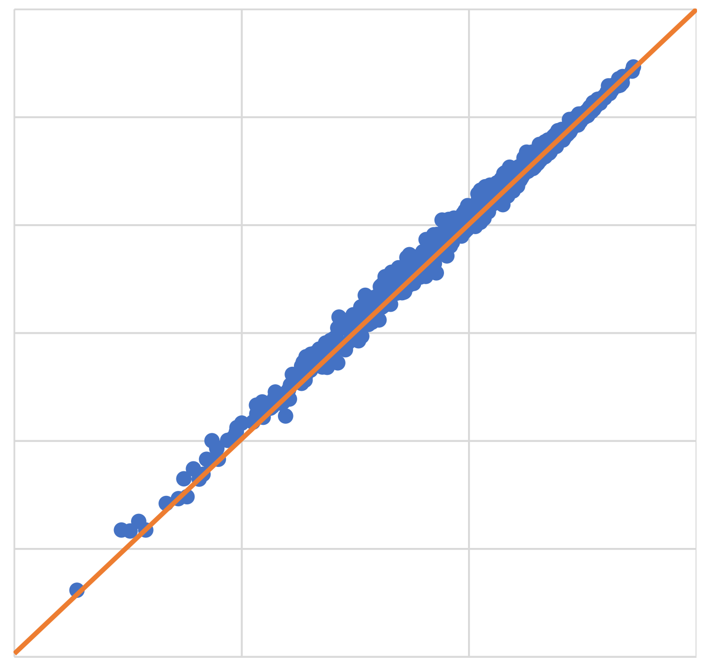
Approach

PVFP from cash flow model dependent on asset allocation

Base vs. allocation 90



Base vs. allocation 116

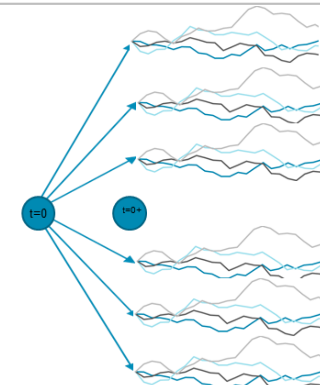


- Variation caused by asset allocations is small compared to effects of stresses of risk factors
- Asset allocations change especially the downside and much less the upside

Approach

Scenarios

- 1‘250 outer scenarios (Sobol sequence) together with 4 inner scenarios each for fitting, thereof 1‘125 for training and 125 for validation
- Combined with 15 asset allocations each (Base allocation + 14 reallocations)
- 75 broadly diversified out-of-sample scenarios



Outer scenarios | inner scenarios

Neural network

- Implementation in KERAS
- Epochs: 500 - 3‘000, activation function: ReLU
- Number of tested networks: ca. 30
- Best results with 5 layers with 8 neurons each
- Target parameter: stochastic PVFP / VIF
- Explanatory variables: durations and average rating (warf*) of 4 bond classes, relative (market value) asset allocations, 17 market risk factors

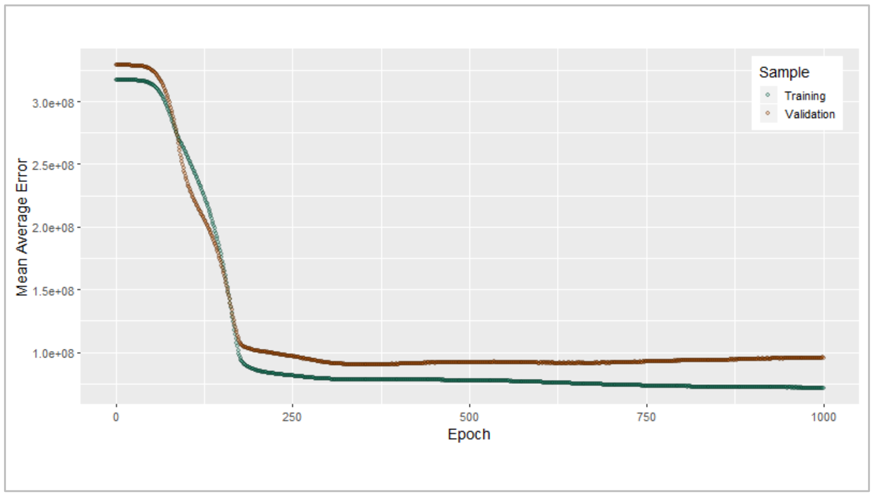
*warf = average weighted rating factor

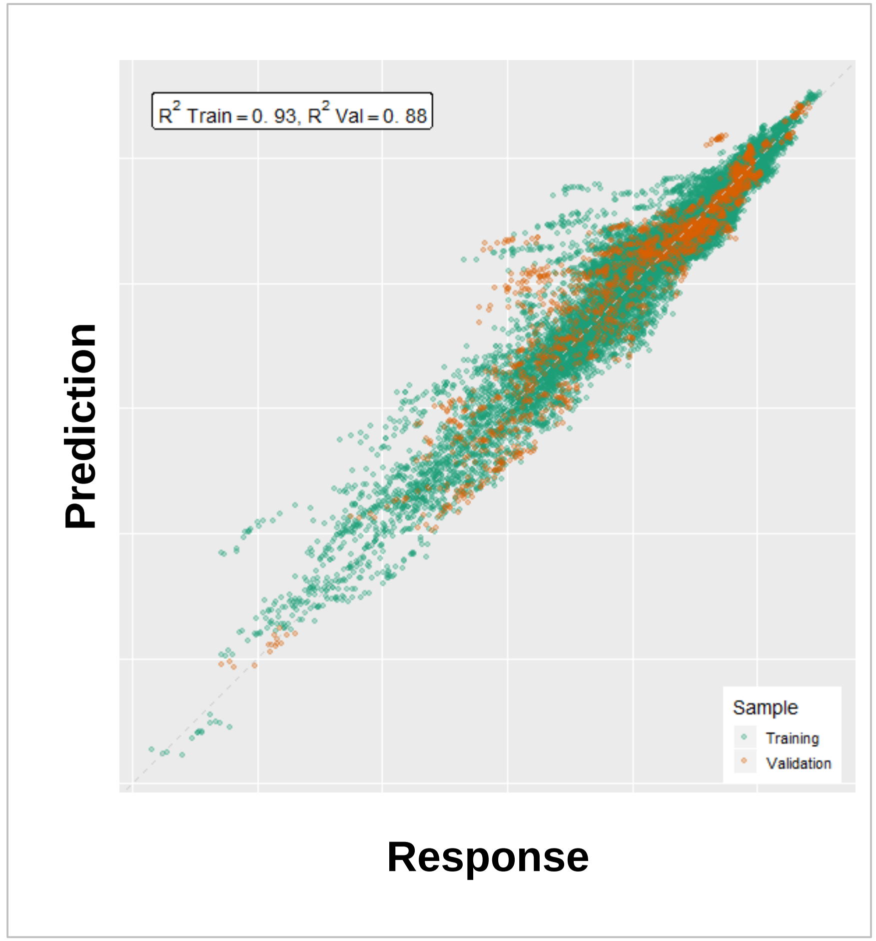
Training results
Goodness-of-fit: In-sample

Best result by a NN with 5 layers with 8 neurons each (trained for 1000 epochs)

Measure of quality:

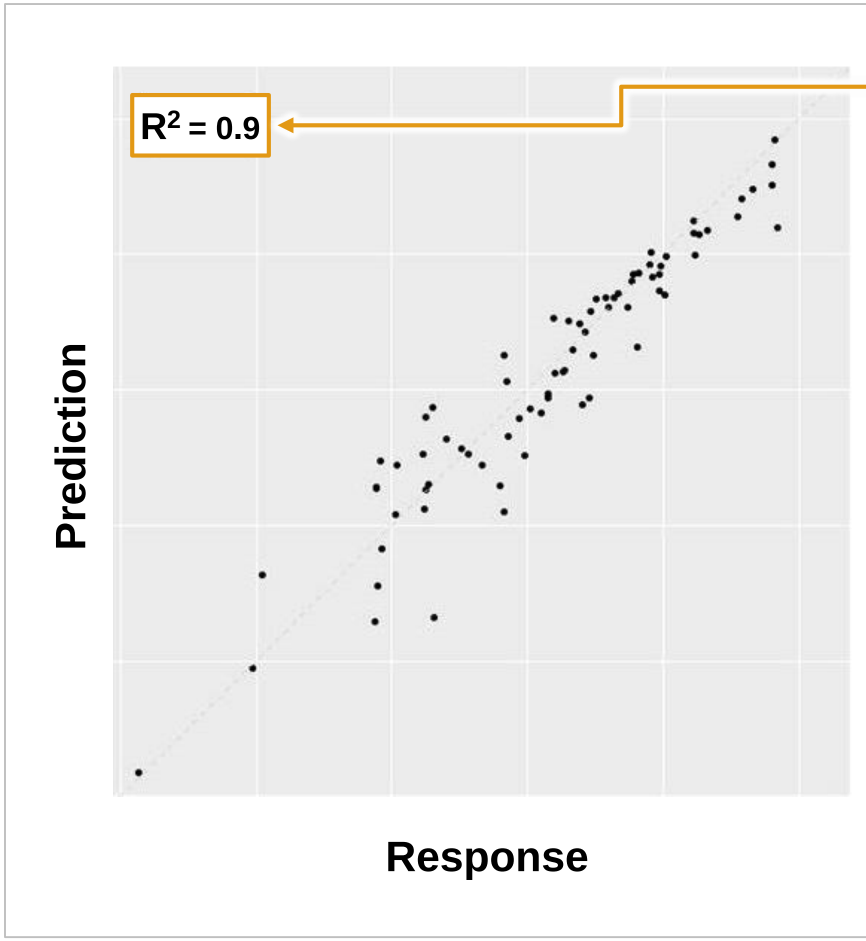
- Quantitative: deviations on training and validation scenarios
- Qualitative: scatterplot In-sample, especially: How do validation scenarios fit to the overall picture?
- Overfitting?





Training results
Goodness-of-fit: Out-of-sample

Out-of-Sample test after optimazition of the hyper parameters



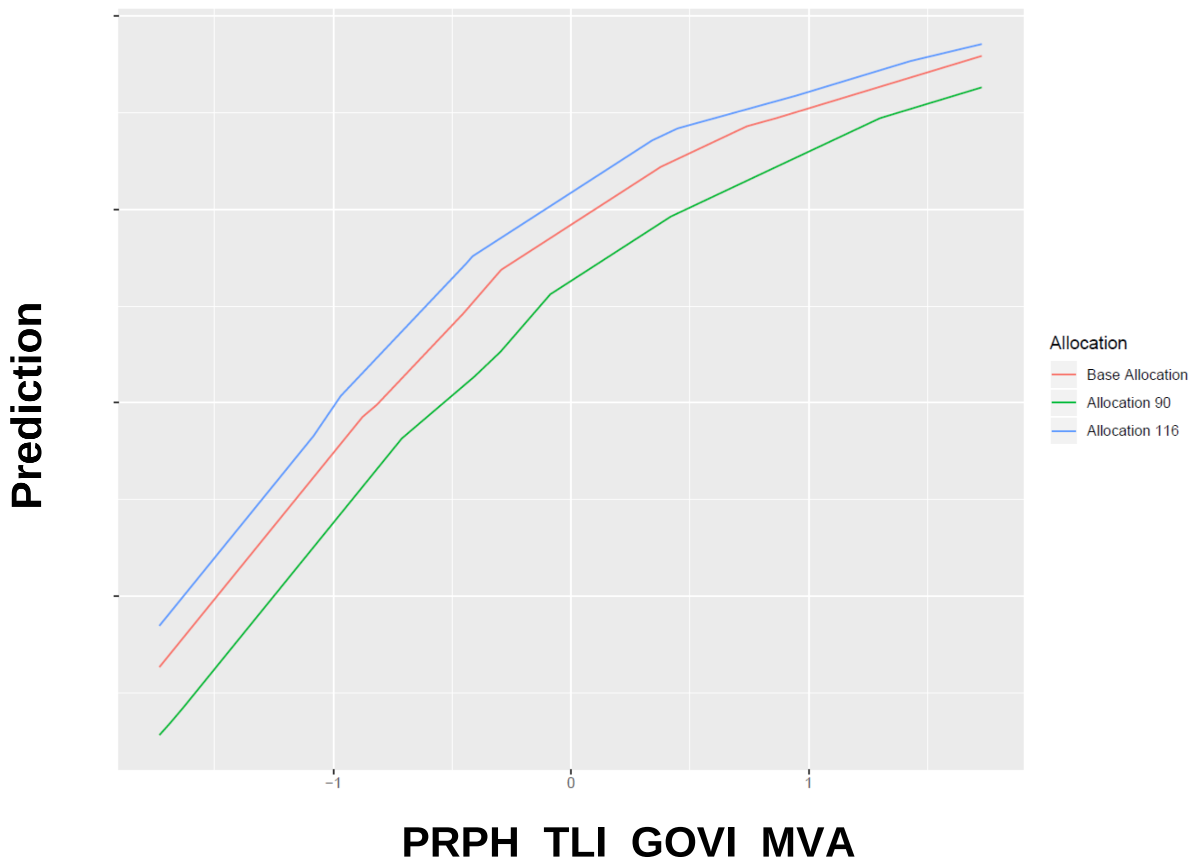
Sufficiently high explanation

Out-of-Sample validation shows final goodness of fit of the NN with respect to

- Interpolation
- Smoothing
  - » Treatment of unknown input signals / risk factor realizations

Training results
Univariate proxy functions

2d plot of the evaluation of the NN by varying the initial market value of the portfolio of government bonds within 3 different asset allocations

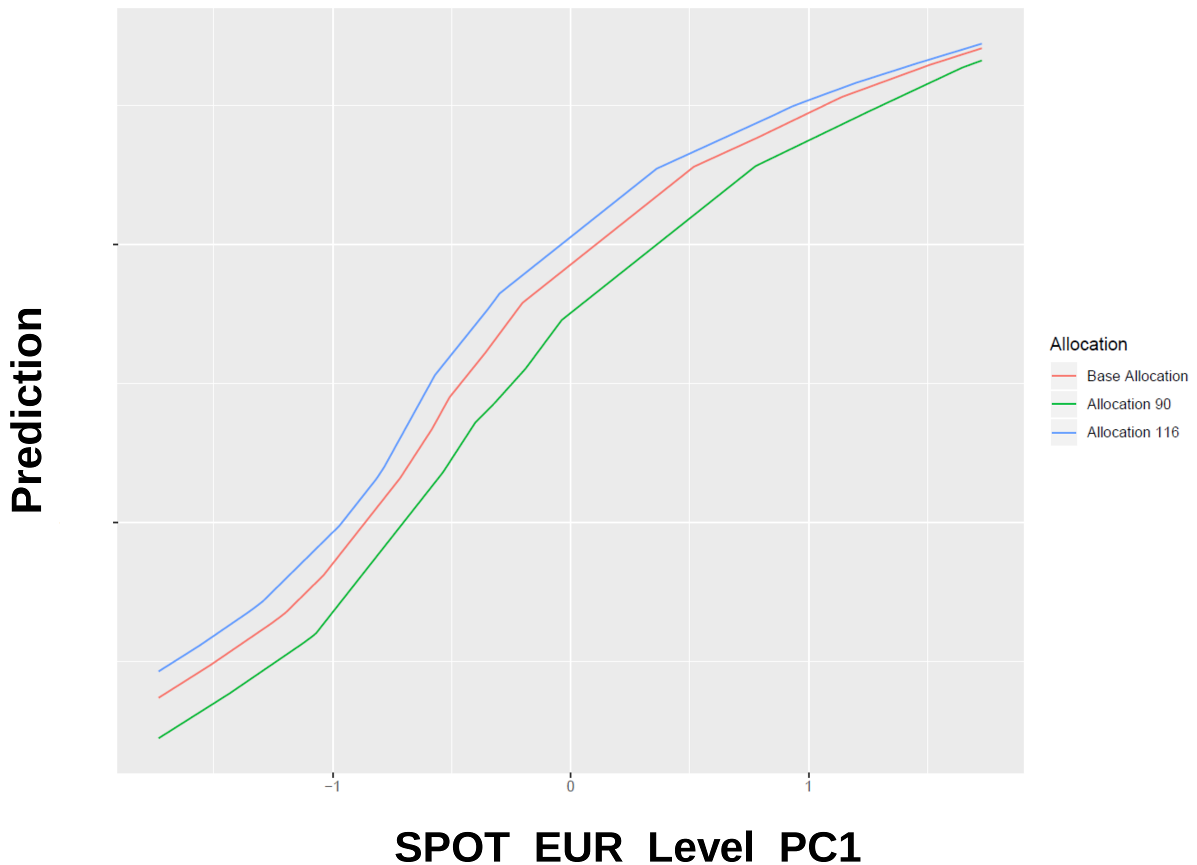


Interpretation

- Increase of own funds the higher the initial market value of government bonds (=decreasing spreads)
- Concave function reflecting the asymmetry of the business

Training results
Univariate proxy functions

2d plot of the evaluation of the NN by varying the initial level of the interest curve within 3 different asset allocations

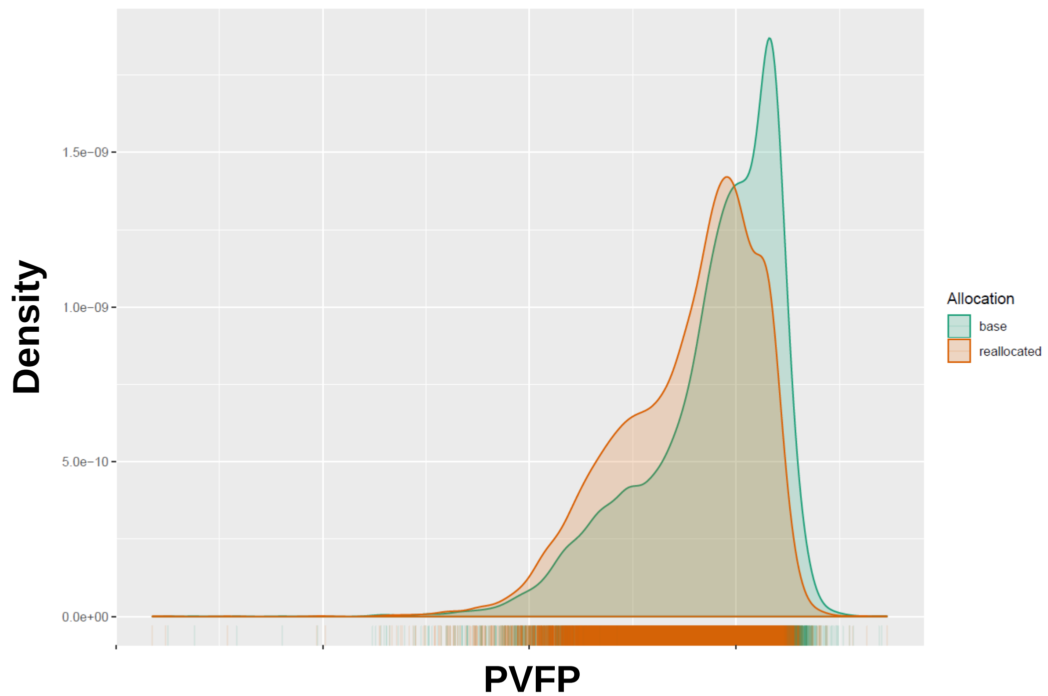


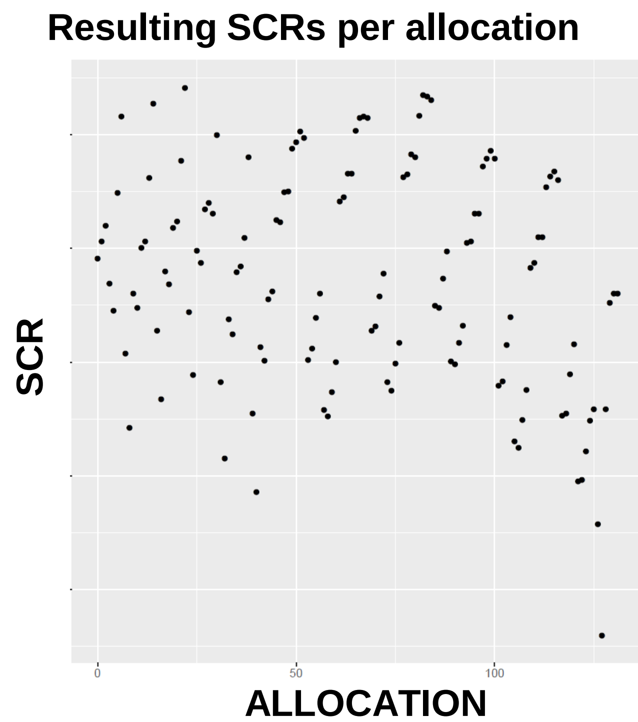
Interpretation

- Increase of own funds if level of interest rates increases
- Concave function reflecting the asymmetry of the business

Applications
Distribution of own funds per asset allocation

The evaluation of the neural net on a multi variate real world distribution of the risk factors results in the distribution of own funds per selected asset allocation (here for 130 examples of asset allocations):

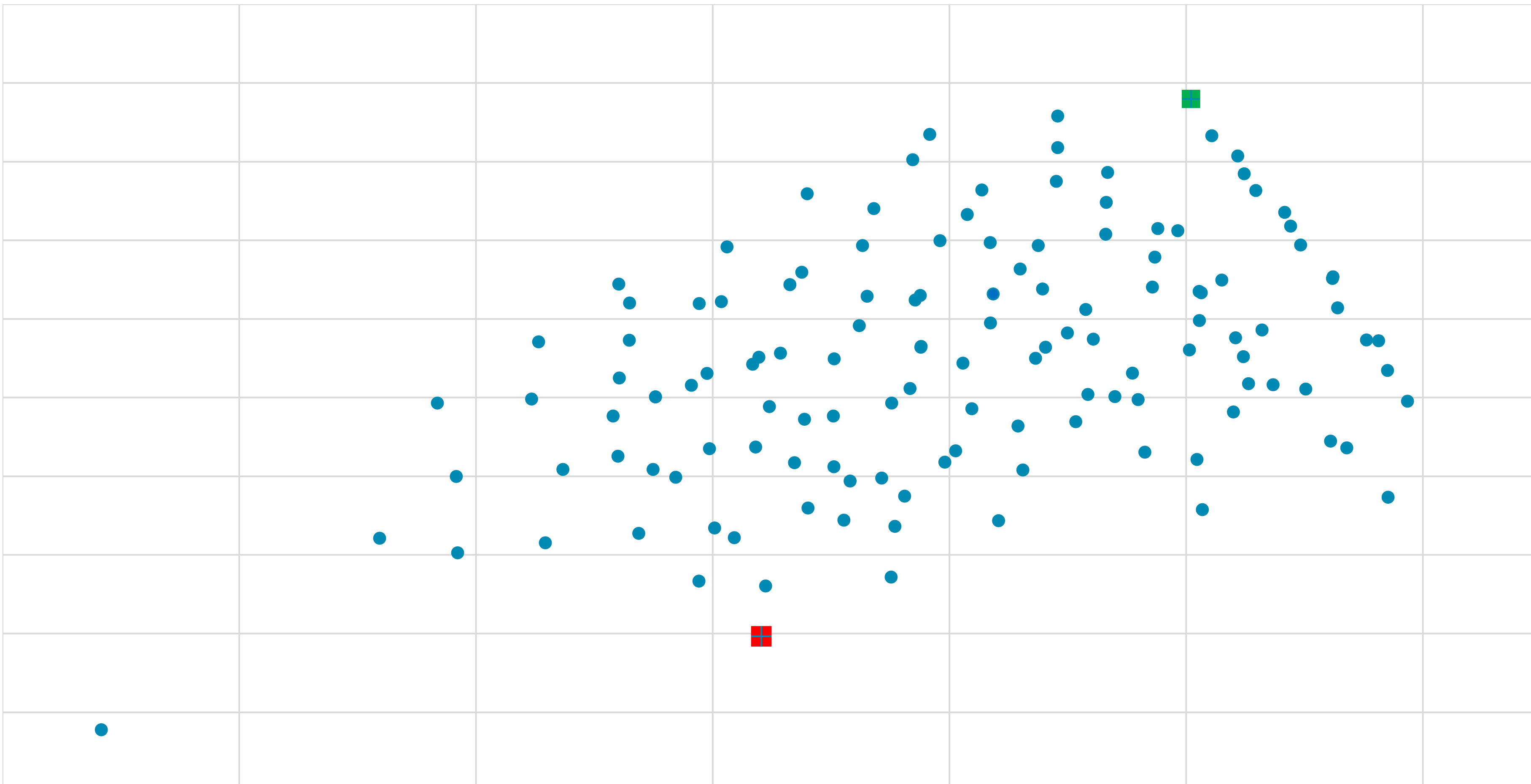




The NN establishes a fast evaluation of the risk profile with many different asset allocations

Applications
Maximising own funds per risk capital

SCR vs PVFP per allocation on a real world evaluation on 130 asset allocations



Questions